Instanton effects in high energy processes.

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Manifestation of the nonperturbative effects in hadron processes at high energy is discussed within the instanton liquid model. Their role in high energy diffractive quark-quark scattering, quark form factor and hard exclusive processes with pion participation is illuminated.

Key words: QCD, Instanton, High Energy, quark, pion

1 Introduction

The very powerful QCD perturbative theory (pQCD) is developed in order to describe the hadron processes at high energies. The scaling and its logarithmic violation are described by the pQCD calculations in the lowest orders of expansion in the strong coupling constant. Coming down in energy more and more powers of the strong coupling constant has to be taken into account. Moreover, in the intermediate energy region the power corrections come into play that are very sensitive to intrinsic hadron structure. Typically, the coefficients of the expansion in the coupling constant and in powers of the momentum transfer are the quark-gluon matrix elements taken at hadronic energy scale that have to be found by nonperturbative methods. These matrix elements are governed by the evolution equations that are determined within pQCD for different hard processes. These equations start to be applicable at momentum transfer squared of order 1 GeV^2 or more where the strong coupling constant becomes small. So, it is necessary to determine the initial data for the evolution equation which is the nonperturbative problem. Another nontrivial situation arises when at high energy two or more hard scales appear. In that case in order to make predictions reliable it is necessary to resum the soft part of the quark-gluon interaction to all orders. Again the presence of the non-perturbative effects may be important in this energy region. There are different approaches to treat manifestation of nonperturbative phenomena at high energies: QCD sum rules, lattice QCD, quark models, *etc.* In this talk we consider few examples of description of the nonperturbative effects in the hadron processes at high energies by applying the instanton liquid model of the QCD vacuum.

Instanton liquid model being effective model of the QCD vacuum describes well the hadrons at low as well at intermediate energies. Thus, contact with perturbative QCD results is possible providing the unique information about the quark-gluon distribution functions in the QCD vacuum and hadrons at low energy normalization point. The various aspects of the instanton induced effects in the high energy hadronic processes had been addressed at the very beginning of the instanton era (see, *e.g.*, [1]). In recent works [2, 3] the interest to them has revived and the hope of direct detection of the QCD instanton induced effects has appeared [4]. We investigate the role of instantons in diffractive quark-quark scattering, their resummation for the quark form factor and as a model for pion bound state describing the distribution amplitudes in hard exclusive processes.

2 Instanton Model of Pomeron (Landshoff-Nachtmann model)

Soft hadronic collisions are described successfully using Regge phenomenology, with the Pomeron exchange being dominating at high energy. The Pomeron is considered as an effective exchange in the t channel by the object with vacuum quantum numbers. That is why the idea that the nontrivial structure of the QCD vacuum is relevant in describing its mechanism.

To illustrate this let us consider high energy diffractive quark-quark scattering, where there is hope that for small momentum transfer the nonperturbative effects give dominant contribution. One of the simplest models of the Pomeron is based on the use of exchange by two nonperturbative gluons. The nonperturbative part of the gluon propagator is given by (in the Feynman gauge)

$$\langle 0 |: A_\mu(x) A_\nu(0) :| 0\rangle = g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} D_{np}(k^2). \tag{1}$$

In the Abelian gauge model considered originally by Landshoff and Nachtmann [5] the nonperturbative gluon propagator $D_{np}(k^2)$ is related to the correlation function describing the gauge invariant gluon field strength correlator (nonlocal gluon condensate). This correlator in general non-abelian case has form

$$\left\langle 0 \left| : G_{\mu\nu}(x) \mathcal{P} \exp\left[ig \int_0^x dz^\alpha A_\alpha(z) \right] G_{\rho\sigma}(0) : \right| 0 \right\rangle = \tag{2}$$

$$= \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left\{ \left(D_0(k^2) + D_1(k^2) \right) k^2 \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \right) + \right.$$
$$\left. + D_1(k^2) \left(k_\mu k_\rho g_{\nu\sigma} - k_\mu k_\sigma g_{\nu\rho} + k_\nu k_\sigma g_{\mu\rho} - k_\nu k_\rho g_{\mu\sigma} \right) \right\},$$

where the first tensor structure is called non-abelian part and the second one is abelian part. Indeed, in the abelian gauge model without monopoles $D_0(k^2) \equiv 0$, and $D_1(k^2) = D_{np}(k^2)$. It is this property that has been used in [5] to relate the Pomeron properties to the value of the gluon condensate.

However, in the non-abelian model one has opposite situation. Really, for the QCD instantons we find [6] $D_1(k^2) \equiv 0$ and $D_0(k^2)$ is nonzero. In the realistic model of the QCD vacuum, where the interaction with vacuum fields of large scale, R, is important, the instanton cease to be exact solution of the equations of motion, but the so called constraint instanton approximate solution (CI) can be constructed [7]. This name is due to necessity to put constraints on the system to stabilize the instanton in the external vacuum medium. It was shown that the constraint instanton has exponentially decreasing at large distances ($\sim R$) asymptotics. The constraint instanton has topological number ± 1 as an instanton, however it is not self-dual field. Thus, in realistic QCD small part of $D_1(k^2)$ appears. Very similar results have been found in the lattice simulations of the gluon field strength correlator [8].

Thus, within the non-abelian models there is no direct connection between the gluon propagator and the gluon field strength correlator. So, let us directly consider the instanton part of the gluon propagator. The Fourier transform of the instanton field is defined as

$$\widetilde{A}^a_\mu(p) = \eta^a_{\mu\nu} p_\nu \widetilde{\varphi}(p^2), \tag{3}$$

where

$$\widetilde{\varphi}(p^2) = \frac{4\pi i}{p^2} \int_0^\infty ds s^3 J_2(|p|\, s) \varphi(s^2), \tag{4}$$

$\varphi(s^2)$ is the (constrained) instanton profile and $J_2(z)$ is the Bessel function. The explicit form of the Fourier transform of the pure instanton solution is well known (in the singular gauge)

$$\widetilde{\varphi}^I(p^2) = i\frac{(4\pi)^2}{p^4}\left[1 - \frac{(\rho p)^2}{2} K_2(\rho p)\right], \quad \widetilde{\varphi}^I(p^2) = \begin{cases} \frac{i(2\pi)^2\rho^2}{p^2}, & p^2 \to 0, \\ \frac{i(4\pi)^2}{p^4}, & p^2 \to \infty. \end{cases} \tag{5}$$

The constraint solution saves its form at short distances, but changes it at large ones:

$$\widetilde{\varphi}^{CI}(p^2) = \begin{cases} \frac{i\pi^2}{4} R^4 I_{CI}, & p^2 \to 0, \\ \frac{i(4\pi)^2}{p^4}, & p^2 \to \infty, \end{cases} \tag{6}$$

where the constant I_{CI} is given by

$$I_{CI} = \int_0^\infty du \quad u^2 \varphi(u).$$

Now, the Fourier transform of the single instanton contribution to the gluon propagator (in the Landau gauge) becomes

$$G^{ab}_{\mu\nu}(p) \equiv \int d^4x e^{ipx} \left\langle 0 \left| A^{a,I}_\mu(x) A^{b,I}_\nu(0) \right| 0 \right\rangle_I = \delta^{ab}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) G(p), \tag{7}$$

$$G(p) = -\frac{4}{N_c^2 - 1} p^2 \int dn(\rho) \rho^4 \widetilde{\varphi}^2(p^2), \tag{8}$$

where the effective instanton density takes into account averaging over the instanton size distribution. Thus, we see again that the gluon propagator and gluon field strength correlator are quite different functions, and the relation between them valid in the abelian gauge model is destroyed in the non-abelian case.

From (5) and (6) it is easy to deduce the asymptotics of the instanton part of the gluon propagator

$$G^I(p^2) = \begin{cases} \frac{(2\pi)^4 n_c \rho^4}{N_c^2 - 1}\frac{1}{p^2}, & p^2 \to 0 \\ \frac{(4\pi)^4 n_c \rho^4}{N_c^2 - 1}\frac{1}{p^6}, & p^2 \to \infty \end{cases}, \quad G^{CI}(p^2) = \begin{cases} \frac{\pi^4 n_c R^4}{16(N_c^2 - 1)} I^2_{CI} p^2, & p^2 \to 0 \\ \frac{(4\pi)^4 n_c \rho^4}{N_c^2 - 1}\frac{1}{p^6}, & p^2 \to \infty \end{cases} \tag{9}$$

Calculating (in very similar way as in the Landshoff-Nachtmann model) at large energy, s, the invariant $\mathcal{T}-$matrix element of the quark-quark scattering exchanging by two gluons we get

$$\langle q(p_3)q(p_4)\,|\mathcal{T}|\,q(p_1)q(p_2)\rangle \underset{s\to\infty}{\longrightarrow} iI(t) \quad \overline{u}(p_3)\gamma^\mu u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2), \qquad (10)$$

with

$$I(t) = \frac{1}{2}\int \frac{d\overrightarrow{k_\perp}}{(2\pi)^2} G\left[\left(\overrightarrow{k_\perp} + \frac{1}{2}\overrightarrow{q_\perp}\right)^2\right] G\left[\left(\overrightarrow{k_\perp} - \frac{1}{2}\overrightarrow{q_\perp}\right)^2\right], \qquad (11)$$

where $G(p^2)$ is defined in (8) with $p^2 \to \overrightarrow{p_\perp}^2$. Except numerical coefficient, this expression is in agreement with the Nachtmann-Landshoff formula. This agreement is due to specific features of the instanton induced interaction.

It is clear from the infrared behaviour of the instanton induced propagator (9) that $I(0)$ (11) is infinite for the pure instanton solution (5), but it is finite for the constraint instanton solution. This fact also noted recently in [2] was one of the arguments to construct constraint instanton that modifies the profile of the instanton at large distances.

From (10) and the optical theorem it follows that the spin averaged total quark-quark cross section is constant at large energy:

$$\sigma_{qq} \sim (n_c\rho_c^4)R^2. \qquad (12)$$

These results have been recently generalized in [2], where the growing part of the total cross section was also found

$$\sigma_{qq} \sim (n_c\rho_c^4)\Delta(t)\ln s. \qquad (13)$$

As was discussed in details in [5] already this simple model of the Pomeron explain many properties of diffractive scattering: the effective vector-like exchange (10), the additive quark rule and the main features of the total cross section (12), (13).

3 Instanton Corrections to Quark Form Factor at Large Momentum Transfer

One of the important questions in the description of hadronic processes is the behaviour of the vertex functions (form factors) at various energy domains. The present section is devoted to the analysis of the instanton induced corrections to the asymptotic behavior of the color singlet quark form factor at the large momentum transfer within the framework of the instanton liquid model of QCD vacuum.

The color singlet quark form factor is determined via the amplitude of elastic scattering of a quark in electromagnetic field:

$$\mathcal{M}_\mu = F_q[(p_1-p_2)^2]\bar{u}(p_1)\gamma_\mu\ v(p_2)\ , \qquad (14)$$

where p_1 and p_2 are initial and final momenta of quark of mass m. The kinematics of the process is described (in Minkowski space-time) in terms of the scattering angle χ:

$$\cosh\chi = \frac{(p_1 p_2)}{m^2} = 1 + \frac{Q^2}{2m^2} \quad , \quad Q^2 = -(p_2 - p_1)^2 \quad , \quad p_1^2 = p_2^2 = m^2 \ . \tag{15}$$

The consistent RG analysis of the total form factor $F_q(Q^2)$ results in the conclusion that its leading large-Q^2 behaviour including all the logarithmic corrections is controlled by the universal cusp anomalous dimension and can be expressed in the following form [9]:

$$F_q(Q^2) = \exp\left[-\int_{\lambda^2}^{Q^2} \frac{d\xi}{2\xi} \left(\ln\frac{Q^2}{\xi}\ \Gamma_{cusp}(\alpha_s(\xi)) - \frac{d\ \ln W_{np}(\xi)}{d\ \ln\xi}\right)\right] \ , \tag{16}$$

where $\Gamma_{cusp}(\alpha_s)$ is the cusp anomalous dimension

$$\Gamma_{cusp}(\alpha_s) = \frac{\alpha_s(\mu)}{\pi} C_F + (\text{Higher} \quad \text{loops}). \tag{17}$$

At high energy the quark receives the eikonal phase during the scattering process off soft gluons. This effect is expressed in terms of the vacuum average of the gauge invariant path ordered Wilson integral

$$W_{np}(C_\chi) = \frac{1}{N_c} Tr\langle 0|\mathcal{P}\exp\left(ig\int_{C_\chi} dx_\mu \hat{A}_\mu(x)\right)|0\rangle \ , \tag{18}$$

In Eq. (18) the integration goes along the closed contour C_χ with cusp.

In the single instanton sector the Wilson integral (18) is in the form:

$$w_I(C_\chi) = \frac{1}{N_c}\langle 0|Tr\exp\left(i\sigma^a\phi^a\right)|0\rangle \ , \tag{19}$$

$$\phi^a = R^{ab}\eta^{\pm b}_{\ \mu\nu}\int_{C_\gamma} dx_\mu\ (x - z_0)_\nu\varphi(x - z_0;\rho) \ . \tag{20}$$

We omit the path ordering operator $\mathcal{P}$ in (19) because the instanton field (3) is a hedgehog in color space. Note, that for the instanton calculations, it is necessary to map the scattering angle to the Euclidean world by the analytical continuation $\chi \to i\gamma$, and perform the inverse transition in the final expressions in order to restore the Q^2-dependence. Therefore, we find that the instanton contribution to the quark form factor [3]:

$$F_q(Q^2) = \exp\left[-\frac{2C_F}{\beta_0}\ln\frac{Q^2}{\Lambda^2}\left(\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(\lambda^2/Q_0^2)}\right) + \ln\frac{Q^2}{Q_0^2}\left(\frac{2C_F}{\beta_0} - B_I\right)\right] \ . \tag{21}$$

results in a finite renormalization of the next-to-leading logarithmic perturbative term.

In the leading order in instanton field the nonperturbative contribution can be expressed in the form (here we use the exponentiation of the single-instanton result in a dilute instanton ensemble [3]):

$$\frac{d\ \ln W^I_{np}(Q^2)}{d\ \ln Q^2} = \frac{\pi^2}{2}\int dn(\rho)\ \rho^4 \ln\left(\rho^2\lambda^2\right) \equiv B_I\ . \tag{22}$$

In order to estimate the magnitude of the instanton induced effect we consider the standard distribution function supplied with the exponential suppressing factor, what has been suggested in [10] (and discussed in [7] in the framework of constrained instanton model) in order to describe the lattice data [11]:

$$dn(\rho) = \frac{d\rho}{\rho^5}\ C_{N_c}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{2N_c}\exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)(\rho\mu_r)^b\exp\left(-2\pi\sigma\rho^2\right)\ , \tag{23}$$

where the constant $C_{N_c=3}\approx 0.0015$, σ is the string tension. Given the distribution function (23) the parameters of the instanton liquid: the mean instanton size $\bar\rho$ and the instanton density $\bar n$ will read:

$$\bar\rho = \frac{\Gamma(b/2-3/2)}{\Gamma(b/2-2)}\frac{1}{\sqrt{2\pi\sigma}}\ ,\quad \bar n = \frac{C_{N_c}\Gamma(b/2-2)}{2}\left(\frac{2\pi}{\alpha_s(\bar\rho)}\right)^{2N_c}\left(\frac{\Lambda_{QCD}}{\sqrt{2\pi\sigma}}\right)^b(2\pi\sigma)^2\ . \tag{24}$$

By using the one loop approximation for the running coupling constant,$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\ \mu^2/\Lambda^2}$, $\beta_0 = \frac{11N_c-2n_f}{3}$, we find the instanton contribution (22) in the form:

$$\frac{d\ \ln W^I_{np}(Q^2)}{d\ \ln Q^2} = -\frac{K}{\beta_0}\pi^2\bar n\bar\rho^4\ln\frac{2\pi\sigma}{\lambda^2}\ ,\quad K\approx 0.74\ . \tag{25}$$

In Eq. (24) we choose the normalization scale μ_r of order of the instanton inverse mean size $\bar\rho^{-1}$. The packing fraction $\pi^2\bar n\bar\rho^4$ characterizes diluteness of the instanton liquid and within the conventional picture its value is estimated to be ≈ 0.12 , if one takes the model parameters as

$$\bar n\approx 1fm^{-4},\quad \bar\rho\approx 1/3fm\ ,\quad \sigma = (0.44\ GeV)^2. \tag{26}$$

The leading contribution to the quark form factor at asymptotically large Q^2 is provided by the (perturbative) evolution governed by the cusp anomalous dimension Thus, the instantons yield the sub-leading effects to the large-Q^2 behaviour accompanied by a numerically small ($B_I\sim 0.01$) factor compared to perturbative one (≈ 0.2). Therefore, the effect of the instantons results in a finite renormalization of the next-to-leading logarithmic perturbative term in the exponentiated expression.

4 Pion Transition Form Factor

Let us consider the pion form factor for the transition process $\gamma^\star\gamma^\star\to\pi^0$ at space-like values of photon momenta. The interest to the pion transition form factor has recently revived due to its measuring by CLEO collaboration [12] at large virtuality of one of the photons. Theoretically, the pion form factor $M_{\pi^0}(q_1^2, q_2^2)$ for the transition process $\gamma^\star(q_1)\gamma^\star(q_2) \to \pi^0(p)$, where q_1 and q_2 are photon momenta, is related to fundamental properties of QCD dynamics at low and high energies. At zero photon virtualities the value of the form factor and its slope (radius) is estimated within the chiral perturbative theory. In the opposite limit of large photon virtualities the leading momentum power dependence [13] of the form factor supplemented by small radiative [14] and power corrections[15] is dictated by perturbative QCD (pQCD).

In this section, we discuss the approach [16, 19] that allow us to match these extremes and describe the intermediate energy region. This approach describes quark-meson dynamics within the effective model, where the quark-quark interaction induced by instanton exchange leads to spontaneous breaking of the chiral symmetry. It dynamically generates the momentum dependent quark mass $M(k^2)$ that may be related to the quark nonlocal condensate [6]. Specifically, we find [17, 18] the pion transition form factor in wide kinematical region up to moderately large photon virtualities and extract from its asymptotics the pion distribution amplitudes (DAs) at normalization scale typical for hadrons.

The invariant amplitude for the process $\gamma^*\gamma^* \to \pi^0$ is given by

$$A\left(\gamma^*\left(q_1, \epsilon_1\right)\gamma^*\left(q_2, \epsilon_2\right) \to \pi^0\left(p\right)\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma M_{\pi^0}\left(q_1^2, q_2^2\right),$$

where $\epsilon_i^\mu (i = 1, 2)$ are the photon polarization vectors. Consider first the low energy region. With both photons real $\left(q_i^2 = 0\right)$ one finds the result

$$M_{\pi^0}\left(0, 0\right) = \frac{N_c}{6\pi^2 f_\pi}\int_0^\infty du \frac{uM(u)\left[M(u) - 2uM'(u)\right]}{D^3(u)} = \frac{1}{4\pi^2 f_\pi}, \qquad (27)$$

where $D(u) = u + M^2(u)$ and $M'(u) = \frac{d}{du}M(u)$, consistent with the chiral anomaly and independent of the shape of $M(k^2)$. Below, for the numerical analysis we choose the dynamical quark mass profile in the Gaussian form $M_G(k^2) = M_q \exp\left(-2k^2/\Lambda^2\right)$, where we take $M_q = 350$ MeV and fix $\Lambda = 1.29$ GeV from the pion weak decay constant, $f_\pi = 92.4$ MeV. We also consider the shape given by the quark zero modes (z.m.) in the instanton field: $M_I(k^2) = M_q Z^2(k\rho)$, where $Z(k\rho) = 2z\left[I_0(z)K_1(z) - I_1(z)K_0(z) - I_1(z)K_1(z)/z\right]_{z=k\rho/2}$, with $\rho = 1.7$ GeV^{-1} being the inverse mean instanton radius and $M_q = 345$ MeV. The mean square radius of the pion for the transition $\gamma^*\pi^0 \to \gamma$ is found to be $r_{\pi\gamma}^2 \approx 1/(2\pi^2 f_\pi^2)$ and is almost independent on the form of $M(k^2)$.

At large photon virtualities $Q^2 = -(q_1^2 + q_2^2)$ the model calculations reproduce the pQCD factorization result ($\omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2)$)

$$M_{\pi^0}(q_1^2, q_2^2)\big|_{Q^2\to\infty} = J^{(2)}\left(\omega\right)\frac{1}{Q^2} + J^{(4)}\left(\omega\right)\frac{1}{Q^4} + O(\frac{\alpha_s}{\pi}) + O(\frac{1}{Q^6}). \qquad (28)$$

The leading and next-to-leading order asymptotic coefficients

$$J^{(2)}\left(\omega\right) \quad = \quad \frac{4}{3}f_\pi\int_0^1 dx \frac{\varphi_\pi^{(2)}(x)}{1 - \omega^2(2x-1)^2},$$

$$J^{(4)}(\omega) \quad = \quad \frac{4}{3} f_\pi \Delta^2 \int_0^1 dx \frac{1+\omega^2(2x-1)^2]}{[1-\omega^2(2x-1)^2]^2} \varphi_\pi^{(4)}(x) \tag{29}$$

are expressed in terms of the light-cone pion distribution amplitudes (DA), $\varphi_\pi(x)$, that are predicted [17] by the model at the low normalization scale $\mu^2 \sim \Lambda^2 \sim \rho^{-2}$

$$\varphi_\pi^{(2)}(x) = \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du \frac{F(u_+,u_-)}{D(u_+)\, D(u_-)} \left[xM(u_+) + (x \leftrightarrow \overline{x}) \right], \tag{30}$$

$$\varphi_\pi^{(4)}(x) = \frac{1}{\Delta^2} \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du \cdot \frac{uF(u_+,u_-)}{D(u_-)\, D(u_-)} \left[\overline{x} M(u_+) + (x \leftrightarrow \overline{x}) \right], \tag{31}$$

where $u_+ = u + i\lambda\overline{x}$, $u_- = u - i\lambda x$, $F(u,v) = \sqrt{M(u)\, M(v)}$, and $\overline{x} = 1 - x$. The parameter Δ^2 characterizing the scale of the power corrections in the hard exclusive processes is

$$\Delta^2 = \frac{N_c}{4\pi^2 f_\pi^2} \int_0^\infty du \frac{u^2 M(u)(M(u) + \frac{1}{3} u M'(u))}{D^2(u)}, \tag{32}$$

Its value is predicted $\Delta^2 = 2.41(2.74)\pi^2 f_\pi^2$ for Gaussian (zero mode) shape of $M(k^2)$, correspondingly. The leading order pion DA, $\varphi_\pi^{(2)}(x)$, is close to the asymptotic form that is in agreement with the results obtained previously in[20, 21]. In the leading order the similar results within the instanton model have been derived earlier in [22].

The asymptotic coefficients $J(\omega)$ may be written in the form [17]

$$J^{(2)}(\omega) = -\frac{1}{\pi^2 f_\pi} \int_0^\infty du u \int_0^\infty dv \cdot$$

$$\cdot \left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \frac{\partial}{\partial z_+} \left(\frac{M^{3/2}(z_+)}{D(z_+)} \right) + (z_- \longleftrightarrow z_+) \right\}, \tag{33}$$

$$J^{(4)}(\omega) = \frac{2}{\pi^2 f_\pi} \int_0^\infty du \int_0^\infty dv v \cdot$$

$$\cdot \left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \left[\frac{M^{3/2}(z_+)}{D(z_+)} + u \frac{\partial}{\partial z_+} \left(\frac{M^{3/2}(z_+)}{D(z_+)} \right) \right] + (z_- \longleftrightarrow z_+) \right\}, \tag{34}$$

where $z_\pm = u + v(1 \pm \omega)$. With the model parameters given above we find for the process $\gamma\gamma^* \to \pi^0$ the values $J^{(2)}(\omega = 1) = 1.83(2.13) f_\pi$ consistent with the CLEO fit $J^{(2)}_{\exp}(1) = (1.74 \pm 0.32) f_\pi$ and the power correction $J^{(4)}(1)/J^{(2)}(1) = 2.97(3.62)\pi^2 f_\pi^2$. The model form factors [17] take into account the perturbative $\alpha_s(Q^2)-$ corrections [14] to the leading twist-2 term with the running coupling that has zero at zero momentum. The perturbative corrections to the twist-4 contribution and the power corrections generated by the twist-3 pion DAs are expected to be inessential.

Thus, within the covariant nonlocal model describing the quark-pion dynamics we obtain the $\pi\gamma^*\gamma^*$ transition form factor in the region up to moderately high momentum transfer squared, where the rapid power-like asymptotics takes place. At larger virtualities the pQCD evolution of the DA slowly goes to the asymptotic limits. From the comparison of the kinematical dependence of the asymptotic coefficients of the transition pion form factor, as it is given by pQCD and the non-perturbative model, the relations between the pion DAs and the dynamical quark mass and quark-pion vertex are derived. The leading and next-to-leading order power asymptotics of the form factor and the relation between the light-cone pion distribution amplitudes of twists 2 and 4 and the dynamically generated quark mass are found.

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